

# Transport Through Quantum Melts

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We discuss superconductor to insulator and quantum Hall transitions which are first order in the clean limit. Disorder creates a nearly percolating network of the minority phase. Electrical transport is dominated by tunneling or activation through the saddle point junctions, whose typical resistance is calculated as a function of magnetic field. In the Boltzmann regime, this approach yields resistivity laws which agree with recent experiments in both classes of systems. We discuss the origin of dissipation at zero temperature.

Two-dimensional (2D) electron systems subject to disorder potentials and external fields exhibit a rich set of quantum phase transitions, indicated by dramatic changes in their transport properties at low temperatures. Here we concentrate on two prominent classes (i) Superconductor to Insulator (S-I) transitions [1-4] observed in a variety of superconducting films and in Josephson arrays, and typically tuned by either disorder or magnetic field. (ii) Analogous transitions in the Quantum Hall (QH) regime: the QH to insulator (QH-I) transition, and transitions between different QH plateaux [5-7].

The longitudinal sheet resistivity  $\rho_{xx}$  in these systems is a continuous function of  $T$ ,  $B$  and  $n$ , the temperature, magnetic field and carrier density respectively. A sharp change in  $\lim_{T \rightarrow 0} \rho_{xx}$  as a function of  $B$  has been interpreted as a quantum phase transition, between localized bosons and localized vortices [1,5].

Recent experiments, however, found a remarkably simple *non critical* behavior of the resistivity which seems to hold in a sizeable portion of the phase diagram.

(i) On both sides of the QH-I transition [8]

$$\rho_{xx} = \frac{h}{e^2} \exp \left\{ \frac{-(\nu - \nu_c)}{\alpha T + \beta} \right\}, \quad (1)$$

where  $\nu = n\phi_0/B$  (with  $\phi_0$  the flux quantum) is the average Landau level filling factor [9], and  $\nu_c$  is its value at the critical point.  $\alpha$  and  $\beta$  are sample specific parameters.

(ii) Near the field-tuned S-I transition [3]:

$$\rho_{xx} = \frac{h}{4e^2} \times \begin{cases} \exp \left( \frac{B - B_{cr}}{\bar{\alpha}T} \right) & \text{large } T \\ \exp \left( \frac{B - B_{cr}}{\bar{\beta}} \right) & T \rightarrow 0 \end{cases} \quad (2)$$

where  $B_{cr}$ ,  $\bar{\alpha}$  and  $\bar{\beta}$  are constants. Note that both Eq. (1) and (2) indicate finite dissipation at  $T = 0$  at all magnetic fields.

It is the purpose of this Letter to provide an interpretation of these resistivity laws using Boltzmann transport

theory of a binary composite of two phases: conducting ( $C$ ) and an insulating ( $I$ ). The primary underlying assumption of this approach, is that *without disorder, the thermodynamic transition at  $T = 0$  is first order*. Mathematically, a crossing of two ground state energy surfaces,  $E_C(B, \mu)$  and  $E_I(B, \mu)$  is assumed. Here  $\mu$  is the chemical potential of the charge carriers. The surface cross at a critical line  $\mu_{cr}(B_{cr})$ . Associated with the two phases are finite size correlation lengths  $\xi_i$ ,  $i = C, I$ . These provide the lower limit to the linear size of an ordered domain.

A smooth random potential  $V(x, y)$ ,  $\langle V \rangle = 0$ , with fluctuation lengthscale  $l_V > \xi_i$  can be incorporated as a local shift in the chemical potential, such that the local energy density is  $e_i(B, \mu - V(x, y))$ . A large  $\langle V^2 \rangle$  breaks the system into domains which are approximately bounded by equipotential contours  $V(x_\mu, y_\mu) = \mu - \mu_{cr}(B)$ . In QH systems, detailed calculations indicate phase separation [10] and domain sizes have been estimated [11].

The first order “quantum melting” assumption is supported by theoretical arguments and some direct experimental evidence.

The theoretical models describing this type of systems exhibit a competition between superconductivity and charge density correlations, as well captured by their mapping to an anisotropic  $XXZ$  pseudospin model on a lattice. Sizeable portions of parameter space for bipartite [12] and frustrated lattices [13], yield first order transitions between solid and superfluid phases [12-14]. Even when the classical transition is of second order, quantum corrections can make it first order [15]. A similar result was found for the Chern-Simons field theory of the QH problem [16].

An experimental evidence for a quantum melt scenario is provided by photoluminescence (PL) data in QH systems [17], which show two distinct modes of relaxation within the sample. These are interpreted in terms of sample inhomogeneity due to binary phase separation.

The assumption of a binary composite structure has been also used to explain non universal critical conduc-

tivity in QH transitions [18], and the quantization of the Hall resistivity at the QH-I transition [19] and in the QH insulator phase [20].

The random potential eliminates the first order thermodynamic transition and produces a second order transition of the transport coefficients which is of a percolative nature [21,22]. This accounts for many universal features observed in different transitions. It can also help explain at least qualitatively, a duality relation observed when  $C$  and  $I$  phases are interchanged across the transition [7,8,23,24].

The primary contribution to the resistivity comes from saddle points of the potential near  $V(x, y) = \mu_{cr}$ . Here we concentrate on the Boltzmann regime, where it is implicitly assumed that incoherent scattering occurs within a single domain size. This requires sufficient zero temperature dissipation, a point we shall return to in the end. Boltzmann theory uses the current density and electric field as classical variables which depend locally on each other. For a finite width distribution of junction resistances in a two dimensional array, the total resistance is given by the resistance of the typical junction [20].

A saddle point junction has two domains separated by minimal distance  $d$ . The Ohmic response depends on the transition rate  $\mathcal{T}$  of the relevant quasiparticles which pass through the junction.

$$\mathcal{T} \sim \begin{cases} \exp\left(-\frac{V''d^2}{8T}\right) & \text{large } T \\ \exp\left(-\frac{S''d^2}{\hbar}\right) & T \rightarrow 0 \end{cases} \quad (3)$$

where  $V''$  and  $S''$  are the curvatures of the potential barrier and tunneling action respectively.

The resistivity of a single junction is given by:

$$R_{xx} \sim \frac{\hbar}{Q^2} \frac{1 - \mathcal{T}}{\mathcal{T}}. \quad (4)$$

In the insulating side of the percolative transition, quasiparticles which flow between superconducting domains are charge  $Q = 2e$  Cooper pairs (bosons), and for QH domains, they are electrons ( $Q = e$ ) in the lowest Landau level.

In the conducting side, the quasiparticles are of vortices or edge quasiparticles which tunnel with rate  $\bar{\mathcal{T}}$  between edges of a narrow superconducting or QH liquid channel respectively. Since a current of vortices produces a longitudinal voltage drop, the channel's resistance is given by the inverse expression to (4)

$$R_{xx} \sim \frac{\hbar}{Q^2} \frac{\bar{\mathcal{T}}}{1 - \bar{\mathcal{T}}}. \quad (5)$$

A recent calculation [25] of the quasiparticle tunneling rate across a quantum Hall strip has found  $S'' = Q \frac{\pi}{4l^2} d^2$  (where  $l^2 = \hbar c/(eB)$ ) for quasiparticles of charge  $Qe$  for the QH liquid. For vortex tunneling through a superconductor there are two limits which depend on the vortex

core dissipation [26]: When dissipation due to the normal core is negligible, vortices obey "Hall" dynamics and  $S'' \approx \pi^2 \rho_s/2$  where  $\rho_s$  is the superfluid density. In the opposite, viscous dynamics limit,  $S'' \approx \eta$  where the viscosity of the normal core is given by Bardeen and Stephen [27] as

$$\eta = \hbar^2 / (2\pi\xi^2 e^2 \rho_n) \quad (6)$$

where  $\xi$  is the vortex core size, and  $\rho_n$  is the normal state resistance measured above the bulk superconducting transition temperature.

In order to compare theory to experimental results (1,2), the typical junction width  $d$  as a function of external magnetic field is required. These can be derived by geometrical arguments. We start with the QH case.

*The QH resistivity law.* We focus on the transition from a  $\nu = 1$  liquid to the insulator [28]. The  $C$  component is an incompressible liquid at  $\nu = 1$ , while  $I$  consists of an electron solid of (lower) average filling fraction,  $\nu_I$ .  $\nu_I$  depends on details such as the disorder potential, and hence is sample dependent [29]. The average filling fraction of the sample  $\nu$  is

$$\nu = p + (1 - p)\nu_I, \quad (7)$$

where  $p$  is the area fraction of the liquid. The percolation threshold in two dimensions is at  $p_c = 0.5$ . The excess area of the majority phase near a saddle point is given by integrating between hyperbolas (see Fig. 1)

$$\delta A = \frac{1}{2} \log(l_V/d)d^2 \quad (8)$$

The total excess area fraction is thus related to the typical  $d$  and  $l_V$  by

$$p - p_c = \pm \gamma d^2, \quad \gamma = N_{sp} \log(l_V/d)/(2A) \quad (9)$$

where  $A$  is total area of the sample and  $N_{sp}$  is the number of saddle points. Using (7), (4), and (3), we find that in both the insulator and liquid sides of the transition  $\rho_{xx}(\nu)$  is given by the universal formula Eq. (1), with  $\nu_c = (1 + \nu_I)/2$ . The constants  $\alpha$  and  $\beta$  give the simplest interpolation formula between the tunneling and activation regimes:

$$\alpha = \frac{8\gamma}{V''}(1 - \nu_I) \quad \beta = \frac{4l^2\gamma}{\pi}(1 - \nu_I). \quad (10)$$

Note that the above analysis does not require extreme proximity to the percolation transition. The crucial assumption is that the solid component of the quantum melt state is sufficiently insulating, such that the transport is dominated by a path that avoids it as much as possible. The same assumption is necessary for observing a quantized Hall resistance, as discussed in [20]. This analysis therefore holds well beyond the critical dynamical scaling regime.

*Resistivity in Field Tuned Superconducting-Insulator transitions.* The picture described above explains the

remarkable similarity of the empirical laws (1) and (2). Both originate from the Gaussian decay of transition rates at the saddle points. For the superconducting side of the field-tuned transition in amorphous MoGe [3], we consider vortices crossing a narrow superconducting channel of width  $d$ .

The effects of internal interactions in the superconductor is provided by the first order line  $\mu_{cr}(B)$ . This allows us to relate the magnetic field to the width of the channel near the percolation field  $B_{cr}$ .

$$\left. \frac{\partial \mu}{\partial B} \right|_{B_{cr}, \mu_{cr}} (B_{cr} - B) = \frac{1}{2} V'' d^2 \quad (11)$$

which yields

$$\bar{\alpha} = \frac{4}{\frac{\partial \mu}{\partial B_c}}, \quad \bar{\beta} = \frac{V''}{\pi^2 \rho_s \frac{\partial \mu}{\partial B_c}}. \quad (12)$$

One can obtain a semiquantitative estimate of  $\rho_{xx}$  for the amorphous MoGe data [3] as follows. At the critical field  $B_{cr}$ , there is a vortex lattice of spacing  $\xi$  in the superconductor. Consider a saddle point channel which is pinched to zero width by two vortices at distance  $\xi$ . As the magnetic field is reduced their touching cores will separate by a distance  $d = \sqrt{C(\phi_0^*/B - \phi_0^*/B_{cr})}$  where  $C$  is a dimensionless constant of order unity and  $\phi_0^* = \frac{h}{2ec}$ . Thus a superconducting channel of width  $d$  is formed. Using the viscosity from Eq. (6), we obtain the zero temperature tunneling exponent, which yields

$$\rho_{xx} \approx \frac{h}{4e^2} \exp \left[ C\pi/2 \left( \frac{\hbar/e^2}{\rho_n} \right) \left( \frac{B - B_{cr}}{B_{cr}} \right) \right] \quad (13)$$

We note that experiments of Ephron *et. al.* [3] have found very good agreement to (13) with  $C \simeq 1.24$ .

*Discussion:* Here we have used Boltzmann theory to explain observed resistivity laws S-I and QH-I transitions. The absence of localization at zero temperature indicates a presence of strong dissipation. This allows us to neglect quantum interference effects at long lengthscales, and justify the use of incoherent Boltzmann transport theory. *However, the origin of this dissipation is not well understood.* One may expect that coupling to gapless Fermi liquid excitations would give rise to dissipation. But how could Fermi liquid excitations be present in S-wave superconductors at zero temperature? “Normal” electrons are recovered in mean field theory where the BCS gap is destroyed by the magnetic field. However, if the *local pair correlations* are present, one might prefer to consider at the boundaries of the *S* domains, a system of quantum disordered Cooper pairs subject to a penetrating field of  $B \approx H_{c2}$ . This field puts approximately one flux quantum per Cooper pair. A flux attachment transforms a Cooper pair into a composite fermion at  $B = 0$  [30]. A metallic state can thus be formed surrounding the *S* domains which could be responsible for the resistive response at  $T = 0$ .

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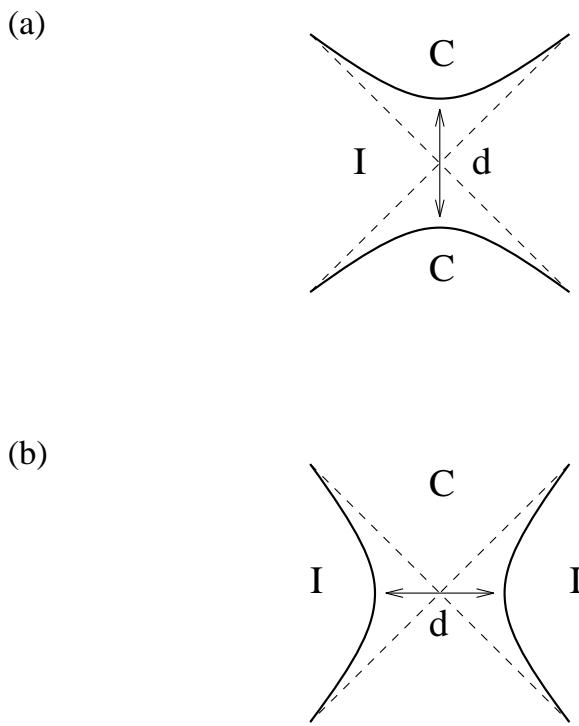


FIG. 1. A typical junction in a C-I mixture (a) in the insulating phase, and (b) in the conducting phase. The thick lines represent the boundaries of the C component, dictated by equipotential contours near a symmetric saddle point of the potential; the dashed lines are the boundaries of C at percolation.